Analyzing Permutation Wordle

MAA MathFest: Combinatorics & Computers Session 2026

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Permutation Wordle

Game:

1. I'm thinking of $\pi \in \mathcal{S}_n$
2. Your goal: guess π in as few guesses as possible
3. After each guess, I will tell you which indices (if any) are correct
4. The game ends when you have guessed my permutation

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Literature:

- *Permutation wordle*. Samuel Kutin & Lawren Smithline (2024)
- *Experimenting with Permutation Wordle*. Aurora Hiveley (2025)
- *Repetition in Permutation Wordle*. Aurora Hiveley (2026)

Example

12345

Example

1**2**345

Example

1**2**345

52134

Example

1**2**345

5**2**1**3**4

Example

1**2**345

5**2**1**3**4

42531

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$$\gamma_1 = \quad 1\mathbf{2}345$$

$$\gamma_2 = \quad 5\mathbf{2}1\mathbf{3}4$$

$$\gamma_3 = \quad \mathbf{42531}$$

Example

$$\gamma_1 = \quad 1\mathbf{2}345 \qquad \begin{cases} \mathcal{I}_1 = \{1,3,4,5\} \\ \mathcal{J}_1 = \{2\} \end{cases}$$

$$\gamma_2 = \quad 5\mathbf{2}1\mathbf{3}4 \qquad \begin{cases} \mathcal{I}_2 = \{1,3,5\} \\ \mathcal{J}_2 = \{2,4\} \end{cases}$$

$$\gamma_3 = \quad \mathbf{42531} \qquad \begin{cases} \mathcal{I}_3 = \emptyset \\ \mathcal{J}_3 = \{1,2,3,4,5\} \end{cases}$$

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$\mathbf{1}\quad 2\quad 3\quad 4\quad 5$

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$$\mathbf{1}\ 2\ 3\ 4\ 5 \implies \mathbf{1}\ 5\ \mathbf{2}\ 3\ 4 \implies \mathbf{1}\ \mathbf{4}\ \mathbf{2}\ \mathbf{5}\ \mathbf{3}$$

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Alt: *CS* has the smallest average number of guesses for $\pi \in \mathcal{S}_n$

Strategy Formalization

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- If $S[3] = 132$, then $514 \mapsto 541$
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$$CS := [(1), (21), (231), \ldots, (2\ 3\ \ldots\ n\ 1)]$$

Implementation with Maple

- There are $sf(n) = \prod_{i=1}^{n} i!$ strategies of length n (yikes!)
- To calculate *average* number of guesses, test a strategy for all $n!$ permutations (brute force)

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Conjecture

Duplicating incorrect information is inefficient

$*$ *We will revisit this idea soon ...*

Statistical Generating Functions

How do we quantify/track a strategy's average performance?

$f_S(x) = \sum_{m \geq 1} a_m x^m$ where $[x^m]f_S(x) = a_m = \#\pi \in \mathcal{S}_n$ guessed in m guesses by strategy S

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Examples from Maple: Cyclic strategies of length $n = 5$

$$x^5 + 26x^4 + 66x^3 + 26x^2 + x$$
$$6x^5 + 27x^4 + 60x^3 + 26x^2 + x$$
$$8x^5 + 25x^4 + 60x^3 + 26x^2 + x$$
$$6x^5 + 27x^4 + 60x^3 + 26x^2 + x$$
$$2x^6 + 10x^5 + 26x^4 + 55x^3 + 26x^2 + x$$
$$8x^5 + 25x^4 + 60x^3 + 26x^2 + x$$
$$6x^5 + 27x^4 + 60x^3 + 26x^2 + x$$
$$x^6 + 10x^5 + 27x^4 + 55x^3 + 26x^2 + x$$
$$6x^5 + 27x^4 + 60x^3 + 26x^2 + x$$
$$11x^5 + 21x^4 + 61x^3 + 26x^2 + x$$

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- Pattern drops here

Additional Maple Computations

From deranged strategies of length $n = 5$:

$$3x^6 + 16x^5 + 27x^4 + 47x^3 + 26x^2 + x$$
$$5x^6 + 9x^5 + 31x^4 + 48x^3 + 26x^2 + x$$
$$5x^6 + 9x^5 + 27x^4 + 52x^3 + 26x^2 + x$$
$$3x^6 + 7x^5 + 31x^4 + 52x^3 + 26x^2 + x$$
$$3x^6 + 15x^5 + 26x^4 + 49x^3 + 26x^2 + x$$
$$2x^6 + 5x^5 + 34x^4 + 52x^3 + 26x^2 + x$$
$$3x^6 + 13x^5 + 28x^4 + 49x^3 + 26x^2 + x$$
$$10x^5 + 36x^4 + 47x^3 + 26x^2 + x$$
$$3x^6 + 13x^5 + 32x^4 + 45x^3 + 26x^2 + x$$
$$3x^6 + 15x^5 + 27x^4 + 48x^3 + 26x^2 + x$$
$$2x^6 + 12x^5 + 30x^4 + 49x^3 + 26x^2 + x$$
$$3x^6 + 9x^5 + 32x^4 + 49x^3 + 26x^2 + x$$
$$3x^6 + 17x^5 + 27x^4 + 46x^3 + 26x^2 + x$$
$$41x^3 + 26x^2 + x + 28x^{\infty} + 18x^4 + 6x^5$$
$$44x^3 + 26x^2 + x + 28x^{\infty} + 15x^4 + 6x^5$$

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Example: $S[4] = 2143$, $\pi = 3421$

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Observations:

- Clearly can happen for non-cyclic derangements.
- True whenever cycle lengths pair off (Hiveley 2026)

Duplication (cont.)

Ex: $S = [(1), (21), (231), (2143), (34521)]$ w/ $\pi = 41523$

$$\begin{array}{ll}
\gamma_1 = 12345 & \mathcal{J}_1 = \emptyset \\
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$*$ Entry 1 is guessed incorrectly in position one in both γ_1 *and* γ_3

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Theorem

CS never duplicates

Intuitively, sure! Proof in Kutin & Smithline

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Every other strategy does

Proof by nasty constructive algorithms (see Hiveley 2026)

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Exercise

Consider a strategy of length $n = 5$ where $S[5] = (2\ 3\ 1\ 5\ 4)$. Show that such a strategy *must* have at least one correct entry on or before γ_3. Show that the same is *not* true for CS.

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Conclusion: Speed and distribution of "correctness" may be more nuanced than we realize.

Thanks & Questions