

Analyzing Permutation Wordle

MAA MathFest: Combinatorics & Computers Session 2026

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Permutation Wordle

Game:

- 1 I'm thinking of $\pi \in \mathcal{S}_n$
- 2 Your goal: guess π in as few guesses as possible
- 3 After each guess, I will tell you which indices (if any) are correct
- 4 The game ends when you have guessed my permutation

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Literature:

- *Permutation wordle*. Samuel Kutin & Lawren Smithline (2024)
- *Experimenting with Permutation Wordle*. Aurora Hiveley (2025)
- *Repetition in Permutation Wordle*. Aurora Hiveley (2026)

Example

12345

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52134

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Example

$$\gamma_1 = \mathbf{12345}$$

$$\gamma_2 = \mathbf{52134}$$

$$\gamma_3 = \mathbf{42531}$$

Example

$$\gamma_1 = \mathbf{12345} \quad \begin{cases} \mathcal{I}_1 = \{1, 3, 4, 5\} \\ \mathcal{J}_1 = \{2\} \end{cases}$$

$$\gamma_2 = \mathbf{52134} \quad \begin{cases} \mathcal{I}_2 = \{1, 3, 5\} \\ \mathcal{J}_2 = \{2, 4\} \end{cases}$$

$$\gamma_3 = \mathbf{42531} \quad \begin{cases} \mathcal{I}_3 = \emptyset \\ \mathcal{J}_3 = \{1, 2, 3, 4, 5\} \end{cases}$$

Cyclic Shift Conjecture

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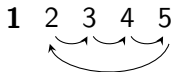
1 2 3 4 5

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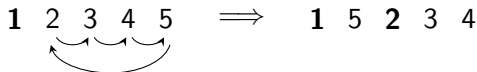


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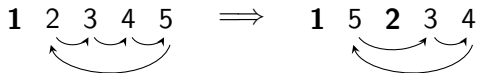


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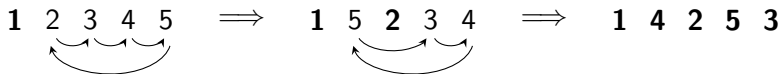


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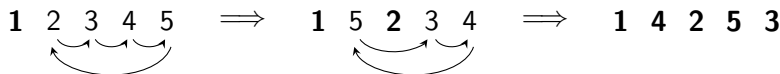


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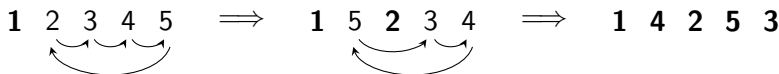
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Alt: *CS* has the smallest average number of guesses for $\pi \in \mathcal{S}_n$

Strategy Formalization

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- $S := [S_1, S_2, \dots, S_n]$ where $S_i \in \mathcal{S}_i$ for all $i \in [n]$
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$$CS := [(1), (21), (231), \dots, (2 \ 3 \ \dots \ n \ 1)]$$

Implementation with Maple

- There are $sf(n) = \prod_{i=1}^n i!$ strategies of length n (yikes!)
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Conjecture

Duplicating incorrect information is inefficient

* *We will revisit this idea soon ...*

Statistical Generating Functions

How do we quantify/track a strategy's average performance?

$f_S(x) = \sum_{m \geq 1} a_m x^m$ where $[x^m]f_S(x) = a_m = \#\pi \in \mathcal{S}_n$ guessed in m guesses by strategy S

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Examples from Maple: Cyclic strategies of length $n = 5$

$$\begin{aligned} & x^5 + 26x^4 + 66x^3 + 26x^2 + x \\ & 6x^5 + 27x^4 + 60x^3 + 26x^2 + x \\ & 8x^5 + 25x^4 + 60x^3 + 26x^2 + x \\ & 6x^5 + 27x^4 + 60x^3 + 26x^2 + x \\ & 2x^6 + 10x^5 + 26x^4 + 55x^3 + 26x^2 + x \\ & 8x^5 + 25x^4 + 60x^3 + 26x^2 + x \\ & 6x^5 + 27x^4 + 60x^3 + 26x^2 + x \\ & x^6 + 10x^5 + 27x^4 + 55x^3 + 26x^2 + x \\ & 6x^5 + 27x^4 + 60x^3 + 26x^2 + x \\ & 11x^5 + 21x^4 + 61x^3 + 26x^2 + x \end{aligned}$$

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- Pattern drops here

Additional Maple Computations

From deranged strategies of length $n = 5$:

$$\begin{aligned} &3x^6 + 16x^5 + 27x^4 + 47x^3 + 26x^2 + x \\ &5x^6 + 9x^5 + 31x^4 + 48x^3 + 26x^2 + x \\ &5x^6 + 9x^5 + 27x^4 + 52x^3 + 26x^2 + x \\ &3x^6 + 7x^5 + 31x^4 + 52x^3 + 26x^2 + x \\ &3x^6 + 15x^5 + 26x^4 + 49x^3 + 26x^2 + x \\ &2x^6 + 5x^5 + 34x^4 + 52x^3 + 26x^2 + x \\ &3x^6 + 13x^5 + 28x^4 + 49x^3 + 26x^2 + x \\ &10x^5 + 36x^4 + 47x^3 + 26x^2 + x \\ &3x^6 + 13x^5 + 32x^4 + 45x^3 + 26x^2 + x \\ &3x^6 + 15x^5 + 27x^4 + 48x^3 + 26x^2 + x \\ &2x^6 + 12x^5 + 30x^4 + 49x^3 + 26x^2 + x \\ &3x^6 + 9x^5 + 32x^4 + 49x^3 + 26x^2 + x \\ &3x^6 + 17x^5 + 27x^4 + 46x^3 + 26x^2 + x \\ &41x^3 + 26x^2 + x + 28x^\infty + 18x^4 + 6x^5 \\ &44x^3 + 26x^2 + x + 28x^\infty + 15x^4 + 6x^5 \end{aligned}$$

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Observations:

- Clearly can happen for non-cyclic derangements.
- True whenever cycle lengths pair off (Hiveley 2026)

Duplication (cont.)

Ex: $S = [(1), (21), (231), (2143), (34521)]$ w/ $\pi = 41523$

$$\begin{array}{ll} \gamma_1 = 12345 & \mathcal{J}_1 = \emptyset \\ \gamma_2 = 54123 & \mathcal{J}_2 = \{4, 5\} \\ \gamma_3 = 15423 & \mathcal{J}_3 = \{4, 5\} \\ \gamma_4 = 41523 & \mathcal{J}_4 = \{1, 2, 3, 4, 5\} \end{array}$$

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CS never duplicates

Intuitively, sure! Proof in Kutin & Smithline

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Every other strategy does

Proof by nasty constructive algorithms (see Hiveley 2026)

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Consider a strategy of length $n = 5$ where $S[5] = (2\ 3\ 1\ 5\ 4)$. Show that such a strategy *must* have at least one correct entry on or before γ_3 . Show that the same is *not* true for CS.

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Conclusion: Speed and distribution of “correctness” may be more nuanced than we realize.

Thanks & Questions

